

ELECTRICAL ENGINEERING

CONVENTIONAL Practice Sets

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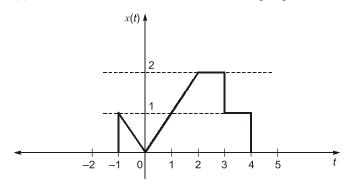
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Continuous Time Signal & System

Q1 For the given signal x(t) as shown below, sketch the following signals.



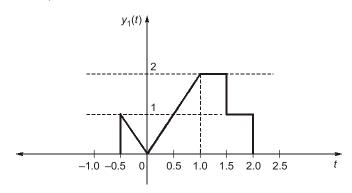
(a)
$$y_1(t) = x(2t)$$

(b)
$$y_2(t) = x(2t + 4)$$

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$$y_1(t) = x(2t)$$
 (b) $y_2(t) = x(2t + 4)$ (c) $y_3(t) = x\left(\frac{t}{2} + 2\right)$

Solution:

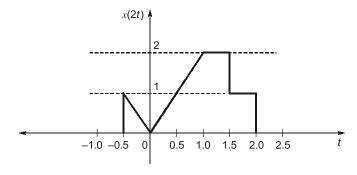
(a) We have to sketch, $y_1(t) = x(2t)$ since, $y_1(t)$ is a 2 times slowed or compressed version of x(t) in time domain. So, the curve is;



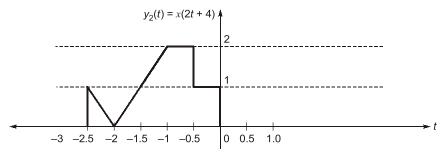
(b) We have to sketch, $y_2(t) = x(2t + 4)$ i.e. $y_2(t) = x[2(t + 2)]$.

So, we can say $y_2(t)$ is the 2 times compressed version in time of a signal which is an advance shift of 2 unit of x(t).

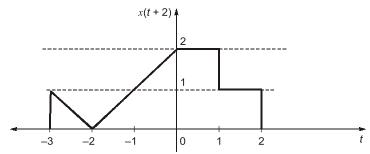
At first we sketch the following:



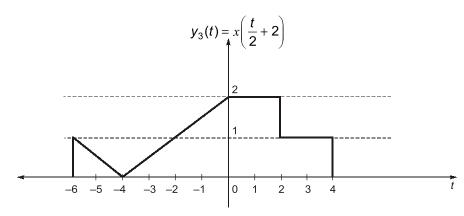
Again, we have to sketch this x(2t) for an advance shift of 2 units means shift the above curve 2 unit in left side as below:



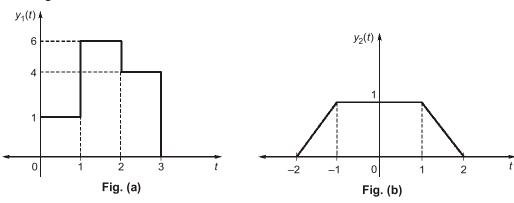
(c) We have to sketch, $y_3(t) = x(t/2 + 2)$. For this firstly, we shift 2 unit in advance shift of x(t) and then expanded this signal x(t + 2), by 1/(1/2) = 2 units of the signal.



Now finally we have to sketch, $y_3(t) = x\left(\frac{t}{2} + 2\right)$ as below:



Q2 For the signals $y_1(t)$ and $y_2(t)$ shown below. Draw the differentiation of the signals and find the equations of differentiated signals.

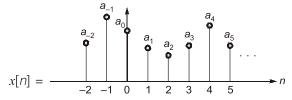


Discrete Time Signal and System

Q1 How can you express a discrete signal as a sequence of a sum of scaled delayed unit sample sequence?

Solution:

Let the discrete signal be given by



Now,

$$\delta[n] = \frac{1}{-1} \frac{1}{0} \frac{1}{1} \frac{1}{2} n$$

Unit sample sequence,

Delayed unit sample sequence,

$$\delta[n-k] = \frac{1}{\sum_{k=1}^{n} k_k k+1} n$$

Thus, the discrete signal x[n] can be expressed as a sum of scaled delayed unit sample sequence.

$$x[n] = a_{-1} \, \delta[n+1] + a_0 \, \delta[n] + a_1 \delta[n-1] + a_2 \delta[n-2] + \dots$$

$$\Rightarrow x[n] = \sum_{k=-\infty}^{\infty} a_k \delta[n-K]$$

Consider a DTS, x[n] is given by $x[n] = 1 - \sum_{K=3}^{\infty} \delta(n-1-K)$. This x[n] can be represented by an unit step function as, $x[n] = u[Mn - n_0]$. Then find the value of M and n_0 ?

Solution:

∴.

Given discrete-time system is,

$$x[n] = 1 - \sum_{K=3}^{\infty} \delta[n - (K+1)] \qquad \dots(i)$$

$$-\infty \qquad 1 \qquad \delta(n-4) \qquad +\infty$$

$$-7 - 6 - 5 - 4 - 3 - 2 - 1 \qquad 0 \qquad 1 \qquad 2 \qquad 3 \qquad 4 \qquad 5 \qquad 6 \qquad 7 \qquad 8 - \dots - n$$

$$x[n] = 1 - [\delta(n-4) + \delta(n-5) + \delta(n-6) + \delta(n-7) + \dots]$$

...(ii)

...(iii)

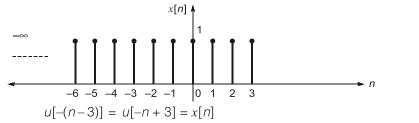
Signals and Systems

Conventional Practice Sets

So, x[n] = 1; for $-\infty \le n \le 3$

As we know that, u[n] = 1 for $0 \le n \le \infty$

and u[-n] = 1 for $-\infty \le n \le 0$



And we have given if x[n] can be represented by a step function as,

$$x[n] = u[Mn - n_0] \qquad \dots (iv)$$

Comparing equation (iii) and (iv) we get,

$$Mn = -n \implies M = -1$$

and

 \Rightarrow

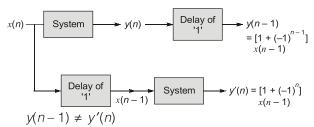
Q3 Consider the system with following input-output relation $y[n] = (1 + (-1)^n) x[n]$, where x[n] is the input and y[n] is the output. Check whether the above system is (a) time variant (b) invertible

Solution:

Given relationship,

$$y(n) = [1 + (-1)^n] x(n)$$

(a) Time invariance test:



Since,

So, the system is time variant.

(b) Invertibility test:

$$\begin{array}{c|c} x(n) & y(n) \\ \hline \delta(n-1) & [1+(-1)^n] \, \delta(n-1) = [1+(-1)^1] \, \delta(n-1) = 0 \cdot \delta(n-1) = 0 \\ 2\delta(n-1) & [1+(-1)^n] \, 2\delta(n-1) = [1+(-1)^1] \, 2\delta(n-1) = 0.2\delta(n-1) = 0 \end{array}$$

Thus, we are getting many to one mapping between input and output. So, the system is non-invertible.

Q4 The step response s[n] of a discrete-time LTI system is given by:

$$s[n] = \alpha^n u[n]$$
; $0 < \alpha < 1$

Determine the impulse response h[n] of the system?

Solution:

::

Step response,
$$s[n] = \alpha^n u[n]; \quad 0 < \alpha < 1$$
 ...(i)

By using the property of discrete-time LTI systems, the impulse response h[n] of the system is given by,

$$h[n] = s[n] - s[n-1]$$

$$= \alpha^{n} u[n] - \alpha^{n-1} u[n-1] = \left\{ \underbrace{\delta[n] + \alpha^{n} u[n-1]}_{\text{Because}, 0 < \alpha < 1} \right\} - \alpha^{n-1} \cdot u[n-1]$$

$$= \delta[n] + \alpha^{n} u[n-1] - \alpha^{n-1} u[n-1]$$

$$= \delta[n] - \alpha^{n-1} u[n-1] + \alpha^{n-1} \cdot \alpha u[n-1]$$

$$h[n] = \delta[n] - (1 - \alpha) \alpha^{n-1} \cdot u[n-1]$$